1) For the following processes obtain the mass balance equations.

i) $F_1 \text{ (m}^3/\text{s)}, \rho_1 (\text{kg/m}^3)$

$F_1 = \frac{h_1}{R_1}, \quad F_2 = \frac{h_2}{R_2}$

ii) $F_i, \rho$

$F = \frac{h_2}{R_2}$
2) For the following reaction we have derived the component and total mass balance equations

\[ \nu_a A + \nu_b B \rightarrow \nu_c C + \nu_d D \]

Show that these five equations are linearly dependent.
(Hint: If \( M_{wi}, \text{kg/kmol} \) is the molecular weight of the \( i^{th} \) chemical species (i= A, B, C, D) then

\[ \sum_i C_i M_{wi} = \rho \quad \text{and} \quad \sum_i M_{wi} r_i = 0 \quad \text{(why?)}. \]

Also remember that the summation operator \( \sum \) commutes with the differential operator \( D=d/dt \) (i.e. \( \sum D = D \sum \)).

3) \( N \) stirred tanks each having a volume \( V \text{ m}^3 \) is connected in series.

Initially all tanks are filled with water and at time \( t=0 \) a salt stream of concentration \( C_0 \text{ moles/m}^3 \) is fed to the first tank at a rate of \( F \text{ m}^3/\text{s} \).

a) Write a transient component balance for a tank \( n \). What type of equation is this?

(The following parts will be attempted after covering transfer function concept in the classroom)
b) Obtain the transfer function relating the output \( C_N \) to the input \( C_0 \).
c) Put a step chance of magnitude \( A \) in \( C_0 \) and obtain the solution for \( C_N \) in the time domain. (You may pursue in a slightly different way then it is thought in the classroom and use the fact that \( L^{-1} \left[ \frac{g(s)}{s} \right] = \int_0^\infty g(\tau)\,d\tau \) when inverting. This will result in integrating by parts \( N-1 \) times.

4) Develop the mathematical description of the process of filling a vessel with an ideal gas. The physical situation is shown in the figure below. Assume that it is possible to deliver a constant mass flow rate of gas \( \rho_f F_f \) to the tank at a temperature \( T_f \) and pressure \( P_f \).

(Neglect kinetic and potential energies).

How would you modify the the result if the gas is non-ideal?
5) A semi-batch distillation unit is charged with 100 mole of a 60 mole % benzene-40 mole % toluene mixture. At any given instant the vapor leaving the still may be considered to be in equilibrium with the remaining liquid. The benzene mole fraction in the exit vapor, y, and the benzene mole fraction in the remaining liquid, x, are related by the expression

\[
y = \frac{2.6}{1 + 1.6x}
\]

Derive an equation relating the amount of liquid remaining in the still to the mole fraction of benzene in this liquid.

The principles of modeling that we have covered in the classroom can be applied to seemingly disparate situations. Here are three examples:

6) The great naval battle of Alisam-i yen in the year 3666 was soon to be joined. Captain Korsanos proudly surveyed his powerful fleet of 33 ships stately sailing in a single file in the light breeze. The good guys’ fleet under Captain Adananos was now in sight, 27 ships strong. As was the custom of naval battles at that time, the two fleets would sail in a single file parallel to each other and in the same direction, firing their cannons madly. Since the rate of destruction of a fleet was proportional to the firepower of the opposing fleet Captain Korsanos was confident that he would win the war. How many ships would Korsanos lose at the end of the battle when the enemy fleet is annihilated? (Note that the actual variables, number of ships, are discrete (integer) you can approximate the system with continuous variables to keep the mathematics simple) (ANSWER: Korsanos has lost 14 ships)
7) While Captain Korsanos was doing his calculations about the outcome of the battle, Captain Adananos was also doing some thinking while sipping his Salgam spirit. After doing some quick calculations he devised the following plan: his fleet would split the enemy fleet, taking on 17 ships first and then attacking the other 16 ships with his remaining ships. Solve quantitatively for the dynamic changes in the number ships of each side as functions of time when this strategy is deployed. (ANSWER: Remaining ships of Adanos is approximately 14 and of Korsanos is zero)

8) Thousands of years have gone by since the war of Alisam-i yen. However, the war sporadically continues between the descendants of Adananos and Korsanos families. In the year 3666 Captain Adananos Jr. is in command of a fleet of 16 starships. Captain Korsanos’s fleet of 20 starships has been spotted approaching. The status of Captain Adananos’s starships is as follows: half of the vessels in his fleet have increased firepower by a factor of two over the firepower of Korsanos’s vessels, which all have the same firepower. The firepower of the rest of Adananos’s fleet is equal to Korsanos’s. But the defensive shields on the second half of his fleet are so designed that these more effective shields reduce by 50% the destruction rate of these vessels by Korsanos’s firepower. Now there are two classes of starships: eight vessels are class A₁, with increased firepower, and eight vessels are class A₂ with improved defensive shields. Assume that half of the Korsanos’s fleet is firing at each class at any point in time calculate who wins the battle and how many vessels of each type survive. Should Captain Adananos be on class A₁ or class A₂ type ship? (Partial answer: better to be on a ship with better shields)