1) A mixing tank is shown below. If mixing occurs isothermally, apply the necessary conservation principles.

\[ F_1 \text{ (m}^3/\text{s)}, C_1 \text{ (kg/m}^3), \rho_1 \text{ kg/m}^3 \]
\[ \downarrow \]
\[ \downarrow \]
\[ F_2 \text{ m}^3/\text{s}, C_2 \text{ kg/m}^3, \rho_2 \text{ kg/m}^3 \]
\[ \downarrow \]
\[ F \text{ (m}^3/\text{s}), C \text{ (kg/m}^3), \rho \text{ kg/m}^3 \]

2) For the reaction \( A + B \rightarrow 2C \) taking place in a batch reactor, starting from the conservation of energy equation, develop an equation which will give the rate of change of temperature with respect to time. Since the reaction is endothermic heat is supplied to the reactor at a rate \( Q \text{ Joules/s (Watts)} \). What is the condition for an isothermal operation?

3) Linearize the following ODE’s.

\[
\frac{dy}{dt} = \alpha y + \beta y^2 + \gamma \ln y \quad \alpha, \beta, \gamma \text{ are constants}
\]

\[
\frac{dy}{dt} = a y - b + c y^n \quad a, b, c, n \text{ are constants}
\]

4) A fluid of constant density \( \rho \text{ (kg/m}^3) \) is pumped into a cone-shaped tank of total volume \( H\pi R^2/3 \). The flow out of the bottom of the tank is proportional to the square root of the height \( h \text{ of liquid in the tank} \). Derive the equations describing the system. If \( F_i \) is a variable, linearize the ODEs you have obtained. Also obtain the linearized ODE in terms of the perturbation variables.
5) The following nonlinear ODE’s are obtained for a variable volume well-mixed heating tank.

\[ \rho c_p h \frac{dT}{dt} = F_f \rho c_p (T_f - T) + Q \]

\[ A \frac{dh}{dt} = \rho Ch^{3/2} \]

Where C, C_p, T_f, \rho, A and Q are constants.

i) Identify the nonlinearities and obtain the linearized equations.

ii) Obtain the linearized model in terms of the perturbation variables that you clearly define.

6) The following ODE’s represent the dynamic behaviour of a CSTR.

\[ \frac{dX_1}{dt} = -\phi X_1 \exp \left( \frac{X_2}{1 + \frac{X_2}{\gamma}} \right) + q(X_{1f} - X) \]

\[ \frac{dX_2}{dt} = -\beta \phi X_1 \exp \left( \frac{X_2}{1 + \frac{X_2}{\gamma}} \right) + (q + \delta) X_2 + \delta u \]

Where X_1 is the dimensionless concentration, X_2 is the dimensionless temperature (controlled variable), and u is the dimensionless cooling jacket temperature (manipulated variable). The parameters appearing in the equations are also dimensionless and have the following numerical values:

\[ \beta = 8.0 \quad \delta = 0.3 \quad \phi = 0.072 \quad \gamma = 20 \quad \text{q} = 1.0 \]

In addition to this, if the dimensionless inlet concentration X_{1f} = 1.0 and u = 0, then the process has three steady states one of which is X_{1ss} = 0.5528, X_{2ss} = 2.7517. Linearize the ODE’s around this steady state.

Note: This problem is taken from Sistu and Bequette "Nonlinear Predictive Control of Uncertain Processes: Application to a CSTR, AIChE Journal, Vol 37, pp. 1711-1723 (1991)
7) Problems 2.3 and 2.10 on pages 38–42 of SEMD (Seborg, Edgar, Mellichamp and Doyle).